The Historical Phases of the Debate on the Transformation of Values into Production Prices

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Abstract

Even though the transformation problem in general does not have a widely accepted solution, it is possible to recognize scientific progress when the historical phases of the debate are put side by side. Therefore, the debate originated from the challenge of the conciliation of the law of value with an equal average rate of profit, though sometimes said to be unfruitful by economists, shows evolution in the long run because it forces Marxist and non-marxist economic schools to confront the quality side of value in theory and to develop abstract models of planned economy in practice.

Key-Words: value and price, transformation problem, labor theory of value, plan and market

JEL Classification: B12, B24, B51

Introduction

The transformation problem of values into production prices is one of the most controversial debates in Political Economy. In the beginning of the 21st century there is still no general accepted solution, a fact which confirms the complexity of the issue. Accordingly, it is common to hear that the discussion suffers from circularity, since it seems to deliver no satisfactory results. In opposition to this view, this paper presents the historical phases of the debate and argues that the discussion achieves scientific progress when one considers its development in the long run.

The main aim of the article is to distinguish the traditional transformation problem from the broader problematic in which it is inserted. So, the advance of the discussion is showed by the confirmation that the traditional approach to the problem is only a specific episode within the discipline of political economy as a whole. In order to defend that idea, three concepts will be analyzed: the definition of the transformation problem, the character of the critique on Marxist economics and the subject under study. If these concepts have suffered positive changes in the last 125 years, the gradual advance of the debate will become evident.

For an adequate analysis of the historical phases of the debate, it is necessary to depart from an abstract presentation. In that sense, the transformation problem describes, first of all, the relation between value and price. So, while the word ‘value’ is a theoretical category in economic science, which lies at a relatively high level of abstraction, the expression ‘price’ is closer to reality. Then, both terms refer to the exchange relation of commodities – value, theoretically and price, empirically. From this perspective, the development from the abstract to the concrete

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2 This conception seems to be the starting point for Marx according to Rojas (1989).
along the three volumes of Das Kapital is a parallel to the transformation of values into prices, and in the ideal tendency, into production prices. The starting point of the discussion is the following question: How does an equal average profit rate come about based on the law of value? Two main methods of answering this ‘challenge’ posed by Engels (1963 [1885]) appeared.

The first method is based on the thesis of the contradiction between Capital 1 and Capital 3, represented by Eugen von Böhm-Bawerk (1974 [1896]). It contains a negative critique of the economics of Karl Marx because it completely rejects Capital as a scientific theory capable of explaining the capitalist mode of production. The solution to the problem, according to this line, is impossible.

The second method of treating the subject goes back to the works of Ladislaus von Bortikiewicz ([1907] 2007). In opposition to the fundamental denial, this path tries to follow the structure of Das Kapital through the formalization and specification of the presentation. Therefore, it allows a more detailed study of the work and represents the continuation of the discussion about the mystery about the equal average profit rate without the intention of leaving the Marxist theory. During the 20th century, different publications on the matter followed this line.

In the flow of the revival of Marxist analysis in the late 60s, the resurrection of the theme was due to Piero Sraffa’s work. Despite the fact that Production of Commodities by Means of Commodities was written to criticize the neoclassical school, it was rapidly discovered that the rehabilitation of Classical Political Economy (Meek (1967)) compelled economic science to confront Marx. But, the use of Sraffa’s model to solve the transformation problem had a contradictory result: the question itself lost its meaning. At that moment, the thesis of the redundancy of the labour theory of value, already anticipated by Engels in the preface of Capital 3, was confirmed and broadly accepted. Therefore there were efforts to end the debate. Surprisingly, the matter continued in a new form: now, the search for the economic meaning of the transformation became the central question.

Given that the discussion is long and does not have a clear solution, it is necessary to organize it historically. This necessity is pointed by Schefold (1979) when he stresses the general incomprehension about the relation between Marx and Sraffa and the difficulty to explain it. By analyzing the publications on the problem it is possible to recognize three historical phases which help to conceive a totalizing presentation for the debate on the transformation of values into production prices. Furthermore, this classification makes clear the Sraffa’s effect over the debate.

The article describes the historical formation of the debate by presenting three development sections, one for each historical phase and a summary table with the main characteristics of each period. A short conclusion recalling the three concepts under investigation closes the paper.

Engel’s challenge

Apparently, the beginning of the history of the debate about the conversion from values into production prices is the correction made by von Bortkiewicz of an eventual mistake of Marx ([1894] 2004). Actually, the debate starts earlier, with the publication of Capital 2.

In the preface, Engels poses the contradiction of Classical Political Economy as a challenge to theoreticians of the time. It consists on the demonstration that ‘an equal average rate of profit can and must come about, not only without a violation of the law of value, but on the very basis of it’ (Engels, [1885] 1963). Engels believed that Marx had already solved the problem. But Marx
(1894) 2004) was not able to clarify the issue entirely, as the forthcoming of alternative solutions showed.

One of these solutions was proposed by Lexis (1895). It is interesting to note that his answer to the challenge anticipates the critique of redundancy of the labour theory of value by more than half a century. According to him, the solution to the enigma of the equalized profit rates could only be achieved if, for the individual commodity, the measure of value as labour time was abandoned. He argued that, from this isolated perspective, there seems to be no comprehensible connection between price and the labour time required to produce the commodity. However, Lexis (1895) is very careful with his conclusions: he does not state that prices cannot be measured and explained by labour time. He admits that real prices can be thought as the point of arrival of the unities of labour time, in a process of transition. This observation calls attention because it points to the implicit function of the labour theory of value in the quantitative determination of exchange relations. So prices can be understood as a form of appearance of labour-values, but no one is obliged to use this point of view. For that reason, Engels ([1894] 2004) describes Lexis as a ‘Marxist disguised as a vulgar economist’.

Engels asserts then that the theory of Karl Marx and vulgar economics explain exactly the same real phenomena, but with different arguments. As a reaction to the thesis of redundancy unconsciously defended by Lexis, Engels simply compares the concurrent propositions to explain the origin of profit. Since one of the aims of Capital 3 was to make Marx’s theory comparable to other economic explanations, this objective seems to have been achieved. Therefore, we can suspect that the argument that the analysis of value made in Capital 1 is unnecessary is the expected conclusion which vulgar economists would arrive when studying Marx’s magnum opus.

Apart from Lexis, other participants tried to contribute to the debate. The way of facing the problem around the end of the 19th century was to describe the operation of the law of value together with the process of competition. The debate centered on the comprehension of the fact that the law dominates the price movements and each participant tried to link the concept of the labour theory of value with the average rate of profit. The labour theory of value, being widely accepted, did not need to be firstly justified, so that the discussion aimed at understanding the theory of value of Marx from the standpoint of David Ricardo.

So the continuity between Classical Political Economy and its Critique are involved in the transformation problem since its original formulation. From then on, the difficulties grew on complexity towards various directions. Engels ([1871-1895] 2003) wanted to demonstrate the existence of the law of value with his controversial ‘historical interpretation’ of ‘Das Kapital’ in order to clarify the reigning confusion. Although for him the solution of Marx was the correct one, there was a great pressure for an explanation of the issue after the publication of book 3. The real economic process behind the equalization of the profit rate based on the labour theory of value had to be better concretized. Therefore, Capital 3 did not solve clearly the paradox.

Those who objected the solution of Marx became both disappointed and satisfied according to Engels ([1871-1895] 2003). For this reason, the difference to David Ricardo needed to be explicitly shown, regarding the determination of the quantity of value. This specific difference would be the ‘technical improvement’ of the Ricardian labour theory of value, not being part of the Critique of Political Economy. This shows that there is an important continuity between classical economics and Marx’s contribution.
According to Engels ([1895-1896] 2003), when economists speak of value, they mean value actually established in exchange. Marx, differently, when speaks of value, means frequently individual value, a quantity which is not defined only in the circulation process, but already in the production sphere. In the context of the transformation, the relation between the spheres of production and circulation originates lots of incomprehension. As Antonio de Paula (2000) rightly asserts, only the dialectical thinking about both sides can clarify how the size of the real exchange value is determined. The connection of individual value and social value occurs exactly in parallel to the linking of production with circulation. Then, the Marxist theory of value has its roots in the formulations of Ricardo and is, in the strictly quantitative sense, a technical improvement. The study object would be, firstly, quantity of value. And, as in Classical Political Economy value can only be expression of labour time, the question on the equal average profit rate based on the law of value is solidly put forward by Marx. The quantitative side of the transformation is seen as solved after Sraffa (1960). Therefore, the third phase of the debate has a very different form than the first one: it is asked about the content of this quantity.

It is known that the quality question was exposed and solved by Marx in the beginning of Das Kapital. This question represents the singularity of the Critique of Political Economy: why the measure of value is expression of labour time? One may note that it is very similar to the problem emerging from the quantitative solution after the ‘Sraffa-shock’: labour can not have a differentiated function when we treat the economy only in its use-value structure. This is exactly the reason why the debate on the transformation problem moves towards ‘the pivot on which a clear comprehension of political economy turns’ (Marx, 1985 [1867]).

In sum, the opposing position to the Marxist school defended that there was an insoluble contradiction between the value system characteristic of Capital 1 and the system of prices presented in Capital 3. The critique of redundancy, unable to establish itself at that moment, but already recognized by Engels ([1894] 2004) as a potential headache, was postponed. Furthermore, no conclusion was formulated. The debate went on and changed after the intervention of von Bortkiewicz (1906). So a second phase emerged, the ‘transformation problem’ as it became definitely internationally known.

The traditional transformation problem

Marx departed from Ricardo in order to overcome the contradiction of Political Economy through the illustration that the law of value dominates the movement of real prices. This means that, although prices are not directly proportional to individual values, there is a mechanism that explains that deviation. However, when trying to present a mathematical formalization for this description, Marx could not arrive at a complete formal procedure. As Heinrich (1999) argues, since then, the problem is not anymore seen as a failure of Classical Political Economy, but as an error of Marx himself.

Marx’s quantitative solution in Capital 3 counts the cost prices in terms of value. This condition guarantees that capitalists can buy the commodities composing the constant and variable capital to their individual values. Even though it is a possible case (if, for example, the organic composition of the respective sectors producing the inputs at question were exactly equal to the average organic composition of the economy), in general, the prices of the commodities

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3 This movement is best expressed in the approach ‘qualitative development’, which is presented in the section of the third phase of the debate.
composing the constant and variable capital will be different from their individual values. Marx ([1894] 2004) recognizes the limitations of his exposition and warns that an ideal formalization should take that into account.  

So the quantitative method had to be modified, so that all possible cases of organic composition could be included in the model. Ladislaus von Bortkiewicz ([1907] 2007), following Tugan-Baranowsky and probably influenced by Wolfgang Mühlpfordt, presents a solution of enormous repercussion in this direction.

The research over the publications on the theme reveals that von Bortkiewicz’s procedure establishes a starting point for the search for an algorithm of converting values into production prices. Accordingly, the quantitative side of the matter is the main area of study of the following articles. The debate is in this phase clearly less ideological and polarized. However, it is not possible to recognize within it a clear path of progress or continuity. This is the reason why the transformation problem seemed to be circular. Indeed, only one aspect united the participants: the problem existed.  

It did not matter if the problem had been artificially created by the modification of Bortkiewicz, if it emerged from the limitations of Marx’s solution, or if it was a product of the ricardian theory of value. The fact is that the question was disseminated in this specific context, which contributed to spread the idea that the transformation problem was only about the formal quantitative conversion of one system into the other.

The phase of the traditional transformation problem is specific in one sense: it has a largely accepted conclusion. This was possible because of the arguments developed on the basis of Sraffa (1960). The result can be summarized like this: the system of values is different from the system of production prices. This difference is due to the criterion of surplus-value distribution. In the system of values, the criterion is the size of the variable capital (the surplus is distributed in proportion to the quantity of living labour in each sector), while in the system of production prices the surplus the sum of the variable with the constant capital acts as the parameter of surplus distribution.

The so-called transformation of values into production prices is the passage from one system to the other. It is formally described through the multiplication of the system of values with a specific matrix, which ‘reorganizes’ the distribution of surplus value so that the production prices emerge together with the profit proportional to the size of each capital, regardless of how it is constituted of variable and constant capital. By the way, the inverse of such matrix allows the inverse transformation, it means, the conversion of production prices into values. This means that there is a mutual quantitative determination between the systems. Now, this result is no longer

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4 Fine and Saad-Filho (2010) also correctly advert that the limitation of Marx’s procedure is real and that he was aware of it.

5 The reference to Bortkiewicz ([1907] 2007) as the initiator of the traditional transformation problem must be maintained because almost all authors of that time entered in the discussion because of his contribution. According to Quaas (1992), Mühlpfordt (1893) had found a formal solution to the mystery of the average equal profit rate even before the publication of Capital 3. Apart from Tugan-Baranowsky and Mühlpfordt, V. K. Dmitriev and the mathematician Georg von Charassof participated in the first formalizations of the labour theory of value. On the origin of the traditional transformation problem, see Howard and King (1992), Quaas (1992) and Schefold (2004). Sraffa frontally criticized the method employed by Bortkiewicz in notes not intended for publication. For further information on this, see Gehrke and Kurz (2006).

6 Except for those who always considered it to be a ‘spurious problem’, like Samuelson.
source of controversies. However, the debates on the significance or interpretation of such transformation seem not to end. Then, after Samuelson (1971) presented his eraser-algorithm, a new controversy arose. Since then, the conflict between Marxists and Sraffians grew. Curiously, Samuelson started being attacked by the Marxist side, faithful to the labour theory of value, as well as by opponents of this theory.

Lerner (1972), for example, criticized Samuelson because he was making ‘illegal concessions’ to the labour theory of value. The theory would be reusable if it was described merely as redundant. Lerner’s panic due to Samuelson’s concessions reveal how the critique from Böhm-Bawerk failed and the labor theory of value became acceptable many years after the complete dissolution of Classical Economics. In the past, the Marxist theory of value had been rejected because it was supposedly wrong. But now, it became logically acceptable. The change from the accusation of contradiction to that of redundancy reveals that Marxist economics gained strength along the development of the debate.

Another reaction to Samuelson (1971) was Southworth (1972). He searches the motives for the increasing interest for Marx among economists and the reasons for their difficulties in comprehending his theory. Part of the confusion would be due to Marx’s own texts, but another part would be the result of different methodologies of investigation. Southworth (1972) asks then if such maneuver made by Samuelson is not a conscientious strategy. This would be similar to that idea raised by Engels, that Lexis was a ‘Marxist disguised as vulgar economist’. Southworth (1972) argues that, in order to oppose the increasing interest in Marxist economic theory, some papers have the political aim of offsetting that movement. Samuelson’s ‘Voila!’ would be an example of such actions, as well as his qualification of Marx, the economist, as a ‘minor Post-Ricardian’.

Samuelson (1971) indeed showed clearly that values could be converted into production prices with help of the already presented method. This enabled the construction of arguments departing from both extremes: from values and from prices. On one side, the analysis of values was said to be an unnecessary detour, on the other side, it was argued that prices were entities without significance if not linked to values. The polarization of the debate became very clear after the dissolution of the traditional transformation problem.

So the algorithm had been found, the problem solved (or extinguished). Still, papers on the matter continued to appear. Why? Because as time passed and the debate grew on formality, the origin of the challenge was lost. By the time a solution was found, the problem was not anymore comprehended in its totality.

Naturally, the debate should turn back to the origins of the problem. And this was exactly what happened. Baumol’s (1974) article represents a new form of facing the issue. He argues that the authors since Bortkiewicz gave too much attention to a theme which was marginal for Marx. ‘What Marx really meant’ was that the distribution of surplus in different forms of income of capital through competition could be illustrated by a mathematical model. Since 1907, the search for such a model had taken all efforts from participants. But what really mattered was to know how the surplus value was distributed in reality. Besides, Baumol (1974) emphasizes that Marx knew that production prices could be quantitatively determined without any mention to values.

7 For a formal summary of the result allowed by Sraffa (1960) see Pasinetti (1979).
Therefore, the crucial movement would be the ‘transformation’ of the surplus value into the different categories of remuneration of capital property, like profit and interest. This conception of the problem differs substantially from the traditional approach.

Up from the point when a mathematical operation of conversion was broadly accepted, new forms of treating the issue began to appear. Since then, many studies look for a meaning of the transformation. In 1977, Ian Steedman published *Marx after Sraffa*. He urges economists to show the necessity and usefulness of the labour theory of value for a materialist analysis of the economy. This officially initiated a new phase.

This means that the traditional transformation problem was only a specific episode inside a broader question involving the continuity and rupture of Political Economy. The practical result of the traditional problem is the definitive refutation of the critique of contradiction between *Capital 1* and *Capital 3* initiated by Böhm-Bawerk ([1896] 2007). The critique of contradiction was thus completely substituted by the critique of redundancy in this particular context.

**The critique of redundancy**

Different alternatives came as an answer to the ‘Sraffa shock’ in order to address the redundancy of the analysis of values. There is disagreement on how to confront this situation. For that reason, the third phase of the debate is fundamentally a dispute between theoreticians of the labour theory of value. This is the reason why the communication with non-Marxists schools became limited in the last years. Among the alternatives of the new scenario are the new solution, the temporal single system, the qualitative development and the probabilistic approximation.

**The ‘New Solution’**

The ‘New Solution’ or ‘New Interpretation’ was the first alternative approach with strong influence. Duncan Foley (1982), Duménil (1983-1984) and Lipietz (1982) were those who put this interpretation forward.

Duménil (1983-1984) argues that the relevance of the labour theory of value is at stake. According to him, the concept of value is a theoretical necessity, because of the aggregation of different use values. So the labour theory of value would have a specific clarifying function: explain the social division of labour in a historical context, while the theory of price could only describe this division in societies that produce commodities. Furthermore, for Duménil, the ‘New Solution’ has to be defended with citations from Marx, so it cannot be freely developed without base from the original text. On the other hand, he recognizes that the interpretation of redundancy can also be demonstrated from Marx’s writings. This would indicate ambivalences in the text that should be carefully analyzed.

Although the ‘New Solution’ seems to be organized, it is not possible to recognize a counter critique to the argument of redundancy at this stage. It was the most popular alternative among labour value theoreticians, but it is being criticized in the last years. Some simply reject it, while others show themselves sympathetic to its objective of defying the dominance of the redundancy solution to the transformation problem. So we may say that the ‘New Solution’ was

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8 This is a representative selection of reactions which aims to illustrate the diversity of interpretations developed from the ‘Sraffa-shock’.
9 Glick and Elbar (1987) and Campbell (1997) are examples of acceptance of the ‘New Solution’.
10 For example Sinha (1997).
pioneer in the political unifying sense, but its fragile theoretical proposition could not be maintained.

The Temporal Single System Interpretation (TSSI)

Another reaction stream is called ‘Temporal Single System Interpretation’ (TSSI). According to Howard and King (1992), it started being developed in the 80s, but it gained popularity only later. It recognizes itself as an alternative to all methods that link values to prices in a simultaneous form. So for proponents of the TSSI, all discussion based on models of simultaneous transformation (including Dmitriev, Bortkiewicz, Sweezy, Seton, Okishio, Morishima, Shaikh, Steedman, and Laibman) cannot explain the meaning of the conversion of values in prices.

In the simultaneous interpretation (the traditional), the transformation does not occur in steps: the value system and the price system are determined at the same time. There are consequently two sets of exchange relations. One known conclusion from this interpretation is the possibility to choose for one of the systems, so that there is no theoretical connection between them. According to Freeman, Kliman and Wells (2004), this led to the separation of the systems and turned the issue into a 'spurious problem'. The redundancy of the value system and the abandonment of Marxian economics were practical results from this perspective.

In the TSSI, on the contrary, the transformation takes place in a chronological order. Firstly, there is the value system, which is determined by the technical conditions of the economy and secondly, the price system is derived from this origin. Then, a temporal connection between both systems emerges, similar to Marx’s formulation. Furthermore, the assumption of the equal average profit rate is studied and it is said that it can only be thought as a tendency. So the TSSI has the objective to put Marx’s own solution in debate in order to clarify the meaning of the transformation. The TSSI actually refutes the correction of von Bortkiewicz with the statement that since then, this erroneous interpretation has dominated the debate.

Authors who criticize the TSSI see in it a further frustrated attempt to ‘save Marx’. Indeed, it is not clear why the TSSI denies the result of the redundancy of the value analysis. For this reason, Gary Mongiovi (2002) affirms that what the TSSI really lacks is a lucid explanation of why we, after all, need the labour theory of value after Sraffa.

More precisely, the TSSI seems to confuse the critique of contradiction with the critique of redundancy. Because of this, it seems to fall again into the trap of trying to justify the labour theory of value. In one word: all efforts of the TSSI are directed to sustain that Marx’s theory of value is consistent. But this is not in dispute anymore! Opponents of Marxian economics argue now, differently from Böhm-Bawerk, that the theory is redundant, precisely because it was shown that it is internally consistent.

The qualitative development

12 For presentations of the TSSI by its proposers, see Kliman and McGlone (1999), Freeman, Kliman and Wells (2004) and Kliman (2007). On my part, I do not think the TSSI is a wrong interpretation, though I believe it employs a wrong strategy for solving the controversy. The task is to recover the Cambridge-Cambridge Critique and to unite Marxian with Sraffian School against mainstream neoclassics. This is my guess on how to reverse the disintegration of Marxian school as described by Kliman (2010).

13 This conclusion is very close to the main argument of Farjoun and Machover (1983). It is not known to me if Farjoun and Machover are in contact with defenders of the TSSI. Both methods are temporal, opposing the traditional simultaneous method.

14 For recent critiques of the TSSI see also Mohun (2003) and Park (2009) and the replies by Freeman and Kliman (2006) and Kliman (2009).
The approximation ‘qualitative development’ was strongly influenced by the pioneer works of Isaak Rubin ([1927] 1978). The authors of this tradition emphasize the qualitative analysis of value in order to solve the dilemma. Here, the concept of abstract labour is studied with great caution. So the qualitative side of the value analysis becomes the focus of attention. The concept of ‘Critique of Political Economy’ becomes central and the difference between Marx and the classical economists are made clear. As a result, the determinations of the quantity of value are neglected. The link between value and price would rather be a development of categories, a connection between expressions on different levels of abstraction, and not a logical-mathematical procedure. The quantitative conversion of values into production prices would be an incorrect method of dealing with the problem, according to this approach.

Why did Marx point out this direction then? Heinrich (1999) argues that, although Marx initiated a new area of investigation, some elements of Classical Political Economy continue to exist in his own writings. This means that Marx still uses classic concepts which would not be compatible with the new discipline.

This argument is similar to that of Paul Mattick (1974), who sees the empirical research on the exchange relations as with low practical relevance. More important than connecting prices to individual values would be the observation of the total fall or rise of the production prices, because that would bring knowledge about the development of the productive forces of the society.

Also from the perspective of the qualitative, Fine and Saad-Filho (2010) deal with the implications of the quantitative solution by arguing that ‘more significant than the algebraic ‘solution’ of the transformation ‘problem’ is the observation that Marx’s theory of value cannot founder on such quantitative conundrums, as the search for a corrected algebraic solution seems to imply’ (Fine and Saad-Filho (2010), p. 112). The crucial point, they argue, is that value exists as a consequence of the social relations. Thus, instead of having to choose between either the value or the price system, Fine and Saad-Filho (2010) claim that the relationship between them must be theoretically recognized and analytically explored.15

The ‘qualitative development’ shelters many other authors and it is one of the most distinct aspects of the third phase. However, it is improbable that this kind of interpretation will be able to end the discussion if the quantitative developments made along the years do not find their rationale. It is important to remember here that the quantitative aspect of the theory of value has strong support from Marx, who was particularly interested in explaining the quantitative relation between value and price. More importantly, dialectics as a method of investigation disallows the focus only either on the quantitative or qualitative aspect of value theory.

The probabilistic approach

From all these alternatives, the probabilistic method deserves attention because it answered to Steedman (1977) most directly. Farjoun and Machover (1983) developed a statistical approach to verify postulates of Political Economy with the purpose to solve the dilemma on the

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15 So they do not reject the quantitative analysis, but call attention to the fact that it must be integrated to some meaning regarding both value and price system. This relationship should clearly determine the differences between the value and organic composition of capital, as Fine (1983) has initially outlined. On the value, organic and technical composition of capital, see Fine (1990). On the consequences of this categorical development of the capital composition for the transformation problem, see Saad-Filho (1997). On the attempt of Ben Fine and Alfredo Saad-Filho to present the dialectics of the quantity and quality in value theory, see Fine and Saad-Filho (2009).
transformation problem. For them, a fundamental assumption of the problem is at the same time a misleading one: the assumption of the equal average profit rate.

They criticize the determinism that emerges from this and present a non-determinist model, where the profit rate can only be given probabilistically. The traditional methods consider that the profits of all sectors are equal because profit rate equalization through competition is a true experience which appears as a reality for the theoretician. Actually, the profit rates are all different, as the empirical results show and the market analysts know.

Despite Marx being aware of this, he treats the profit rates as if they had equalized. The problem of observing the equal average profit rate as something real lies in the inversion of the transformation. Values can then be derived from prices. Marx warned that the assumption of the equal average profit rate could lead to this erroneous conclusion and for that stressed that, even when the assumption is made, there is only one possible direction of transforming, which is that from values towards prices. The probabilistic approach can be summarized as follows.

The exchange relations on the market are adjusted to the production prices through the process of free market. The price of a commodity can be freely negotiated between buyer and seller, but the variability of prices are limited by unknown parameters. Prices of production are not the object of study in this approach, in opposition to Sraffa: there is only the value system (individual values in production) and the market prices (observable exchange relations formed on the market). Marx writes that the concrete exchange relations are governed by economic laws, specially the law of value. This means that, even though the market prices are ‘open’, the law of value dominates and regulates them. The connection between value and price can be then modeled with probabilistic calculations where there is no absolute tendency for equilibrium.

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**Figure 1: values, prices of production and market prices**

Considering this summary of Farjoun and Machover’s approach, it is possible to comprehend the meaning of the traditional algorithm. The starting point of this presentation is the value system (box labeled values). These are the individual values of commodities. In other words, they are the individual labour time used to produce each individual product. This system represents the technical reality and it is inaccessible information in societies which socialize

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16 Each box represents a system. So ‘values’ represents the value system, ‘prices of production’ represents the production prices system and ‘market prices’ are the empirical exchange relations which really took place.
individual labour through the market. It is determined by the own technical structure given by the productive forces of society. This combination of values does not allow in reality an equal profit rate for capitals with same size but with different organic compositions.

The traditional solution is represented by the double arrow ‘a’: it is an algorithm that enables a new distribution of the surplus value according to the equal average profit rate. Accordingly, the profit rate in Sraffa’s model and the production prices are determined by the use-value structure of production. This solution must be carefully interpreted: as there is reciprocity between both systems, it is said that values are not the starting point towards production prices. But, as circulation is not yet in picture, it is clear that on this stage, one is not dealing with concrete exchange relations.

Alternative ‘b’ represents the probabilistic method. The adjustment of values to production prices takes place only trough a trial-and-error process typical of the market. Therefore, production prices and the average profit rate can only be an abstract creation, an objective around which the production finds orientation. The transformation ‘b’ goes from the value system towards the market prices system without a deterministic calculation. Moreover, market prices are the only way to get access to technical reality of the private productive unity. In other words: technical conditions of production, under capitalist relations, show themselves only through the market process. Production data (input quantity, including labour) are private property in the capitalist mode of production and are socialized only afterwards on the market. Although these data exist, we do not see them prior to circulation. These data would be visible beforehand only through nationalization and/or planning. Therefore, market prices act as empirical information about the technical conditions of production in capitalism.

In this sense, ‘value’ and ‘price’ correspond to those two levels of observing exchange relations. Accordingly, values are not only theoretically but also temporally prior to prices, as Marx argued. Bettelheim (1969) remembers that the differences between the price theories and the Marxian framework are not visible on the practical level of economic calculation. This supports the idea that economic schools away from Marx, when dealing with the transformation problem, may indirectly produce technical material for the praxis of planning.

The relation between ‘a’ and ‘b’ becomes clear with the passage from the production price system to the sphere of circulation. ‘a’ would be the theoretical transformation, representing the algorithm of conversion which expresses the result of competition. This was the focus of the traditional transformation problem. ‘b’, on the other hand, would be the transformation on a more concrete level.

Prices of production represent the objective of the movement and the assumption of the equal average profit rate: after the market prices are formed on the circulation sphere, this system is compared with the system of production prices. If they are different, an equal average profit rate was not achieved. This is the reason for a new allocation of factors in the next period. The system of values will transform itself in the new system (values’). The importance of this constant comparison and reallocation is expressed in the relevance of the price signs, i.e., in the relation between supply and demand.

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17 This is the starting point for the formalization of the critique of redundancy developed by Samuelson (1971).
18 This is the reason why economic schools in general are led to build planned economy models when dealing with the transformation problem. For a Marxist position see, for example, Cockshot and Cotrell (1989), who currently develop Farjoun and Machover’s approach further and link the results with the economic calculation debate.
This should partially indicate how the economic calculation debate and the operation of the law of value in the planned economy can be put in connection to the traditional transformation problem.

A summary table

The three historical phases of the discussion are displayed side by side, so that a simple comparison can be made. The organization of the table does not imply that the themes are strictly separated. It means that each period had specific characteristics.

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<td>How does an equal average profit rate is formed based on the law of value?</td>
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Figure 2: The historical phases of the debate on the transformation of values into production price

19 The ‘Sraffa’s effect’ describes what was the influence of *Production of Commodities by Means of Commodities* on each phase.  
20 The ‘Formal question’ describes the official starting point for dealing with the transformation problem. For phase 3 there is still not such question because the problem poses itself as a ‘spurious problem’. It is suggested that a formal question should be formulated in the sense of relating the law of value with real problems of economic planning. My suggestion is to ask: How is the law of value related to economic planning?
The Progress of the Debate

According to the present study, the traditional transformation problem lies within a greater problematic involving the questions of continuity and rupture in Classical Political Economy. With respect to those three concepts under analysis, three results can be here summarized.

First, it is necessary to broaden the definition of transformation of values into production prices, since the debate deals with a vast field of analysis in theoretical political economy. Therefore, a strict definition of the theme makes only sense when one wants to observe a specific phase of the debate. The classical conception of the transformation is related exclusively to the phase of the 'traditional transformation problem', which encompasses only the problem of quantitative conversion.

Second, as time passes the emphasis on certain aspects of the issue is different. Mainly, the form of the critique on Marxist economics suffered a significant change. From the perspective of theory, the progress is expressed in the failure of the Böhm-Bawerk-type of attack. Here we have an important result. Because the argument opposing Marxist economic theory has moved from an accusation of 'contradiction' to one of 'redundancy', it becomes very visible that any rejection of the economic theory presented in Capital is a matter of political choice, not of technique. The debate was strongly polarized after Samuelson (1971) precisely because of this.21

Third, the analysis of the value form became the central subject under study after the implications of the model of Sraffa (1960). At this moment, the economic meaning of the transformation started to gain attention. Now, old quantitative solutions are opening space for the formulation of a new questioning of qualitative nature. As the content expressed in the relations of quantitative exchange became empty after Sraffa, the discussion was directed to the qualitative analysis of value.

This last point demands perhaps a more detailed explanation, since it may be the most fragile of these three results. As Schefold (1974) had warned, the Critique of Political Economy encounters tough resistance every time the theoreticians are compelled to deal with the qualitative side of the theory of value. The orthodox economic theory systematically hinders the study about the quality of value by concentrating all efforts on the quantitative side. Because of this, even though the contemporary phase of the debate on the transformation problem calls for the qualitative analysis of value theory, economists distant from the Marxist tradition will abandon the debate by (correctly) arguing that it is useless for the quantitative measure of wealth. This is the context in which the critique of redundancy of the labour theory of value should be understood. In the turn to the 21st century, there can be no discussion similar to the phase 1 because today, the law of value is not generally accepted as a subject of study in economics. In the end of the 19th century there were no doubts that values needed to be somehow linked to prices since the labour theory of value was broadly accepted before the dissolution of the ricardian school. Moreover, if the challenge posed by Engels is accepted, one cannot affirm that the redundancy-argument is a solution to the problem, since it is not based on the law of value. In this sense, it would be merely an answer to the traditional quantitative problem.

21 This interpretation of the controversy on the labour theory of value is thus very different from that of Kliman (2007). After Sraffa, the critique of contradiction is exploded and the argument of redundancy, which is not refutable, is employed politically by reactionaries as a way to maintain Marx distant from the official teaching of economics. The change of the critique from ‘contradiction’ to ‘redundancy’ was a positive result for the Marxian theory of value.
On the other hand, even though non-marxist (or still Marxist) schools do not consider the labour theory of value, they develop abstract models of planned economy when they conciliate both the value and price systems. By doing that, they unconsciously contribute to a better understanding about the capitalist economy and to the development of the theory of mixed economies. One of these results was the development of approximations that, in combination to input-output models, may offer new tools for the practice of planning.\(^{22}\)

So, as the quantitative problem is solved, the problematic must necessarily be directed to the Critique of Political Economy as a way to understand the concept of value for Marx. And since value for him is very closely related to abstract labour, the debate on transformation problem needs to comprehend how did this category was historically formed. But this must not neglect the aspect of the quantitatively concrete determination of prices, since it has practical validation in the field of planning. Only then economists will fulfill their scientific objective of treating the theory of value dialectically.

Finally, it is shown that, due to the changes in the definition of the problem, in the character of the critique and in the subject under study, there is scientific progress in the debate. In this context, it is possible to comprehend that, even on an unorganized framework, the debate on the transformation problem is moving to the analysis of the value form and to the theory of possible economic planning within the capitalist society.

**References**


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\(^{22}\) A general presentation about that process was made by Clark (1984).


